EXECUTIVE SUMMARIES

The Executive Summary consists of a brief description of the problem followed by a largely equation-free summary of the progress made and the results obtained by the Study Group.

MODELS FOR HONEYBEE ARRIVAL AND BLOSSOM PHENOLOGY

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Executive Summary

The aim of the Study Group was to investigate the mismatch in phenology of honeybees and blossoms which is occurring as a result of climate change. Particular attention was paid to the effect of the advancement in phenology of honeybees and blossoms on bee population numbers. This is a complicated, multi-faceted problem that requires an in-depth understanding of both blossom and bee behaviour and how these two different groups interact.

As a start to this investigation, blossom phenology and the advancement in flowering dates was studied. Initially, a simple model for blossoms that is independent of bee behaviour was developed. Three phases in one blossoming cycle were considered: acceleration, saturation, and decay. The various stages in this cycle were accurately described using a beta distribution. An in-depth knowledge of the complicated biological processes which occur was not required to obtain a reasonable understanding of blossom behaviour.

A compartment model was then developed to model a single honeybee hive. Two sub-populations were identified: hive bees and forager bees. Hive bees

are responsible for maintaining the hive whilst forager bees serve the hive by replenishing the food supply. Forager bees have a much higher death rate and are vulnerable to elimination. The effect of blossoms on bee numbers was incorporated through a coupled system of differential equations describing the available food supply in the hive. This highly nonlinear system of ordinary differential equations was solved numerically.

The next step in this project is to now incorporate the effect of the advancement in phenology of bees and blossoms to predict the behaviour of honeybees in the future.

MODELLING THE TURBULENT FLOW IN LAKE KIVU

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Executive Summary

Lake Kivu is a lake in Africa which is situated on the border of the Democratic Republic of Congo and Rwanda. It is a very deep lake (maximum depth 475 m) which causes stratification into roughly three layers: a biozone layer with depth 50-80 m measured from the surface, an intermediate layer which extends from the biozone layer for approximately 200 m in depth, and then a deep layer which extends from the intermediate layer to the bottom of the lake. Communities from both countries living on the lake are dependent on the fauna found in the biozone as well as the dissolved methane in the deeper layers which is extracted and used for electricity generation. It is also one of three lakes in Africa which experience limnic eruptions and previous Study Groups considered the effects of gas inflow at the bottom of the lake due to nearby volcanic activity that may lead to such eruptions. Due to the dependence on the biozone by communities and the economic and industrial gain from gas extraction in the lake, a deeper understanding of the dynamics of the lake are crucial. The Study Group was tasked with modelling turbulent flow in Lake Kivu and understanding the effects it may have.

The Study Group considered an existing model of surface layer turbulent flow

originally developed by Jörg Imberger [1] with the idea of adapting it to Lake Kivu. The Study Group divided into three subgroups: one that would focus on understanding the existing model, one that would numerically solve the nonlinear ordinary differential equations derived by Imberger, and one that would obtain data about Lake Kivu to determine the parameters defined in Imberger's model.

The Study Group was able to derive all of the equations of motion of Imberger's model, compute most of the parameters in Imberger's model for Lake Kivu and solve the given ordinary differential equations numerically with simplifying assumptions for some idealistic, non-real initial conditions. The preliminary results indicated that wind speeds during monsoon season would potentially generate enough shear surface stress for the flow to become turbulent. This turbulence, however, would be weak.

Future work would include nondimensionalization of the equations of motion, the numerical solution of the nonlinear ordinary differential equations without simplifying assumptions and adaption of Imberger's model taking into account the volcanic activity close by Lake Kivu. The volcanic activity introduces carbon dioxide and methane gas into the deepest layer of the lake which could be modelled as an influx into the base shear layer of Imberger's model.

References

[1] Imberger, J. Environmental Fluid Dynamics: fluid processes, flow scales and processes and equations of motion, Academic Press, New York, 2013, pp 278-287.

JUICE HOLDUP DETECTION IN A SUGAR CANE DIFFUSER

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Executive Summary:

In order to efficiently remove sugar from crushed and shredded cane in a sugar cane diffuser, it is necessary to maintain the water level within the cane bed. If the water level is too high, then flooding occurs, leading to shortcutting of the desired recycling of the flow within the diffuser. If, on the other hand, the water level is too low, then sugar will not be efficiently removed from the upper parts of the cane bed. Maintaining the water level at the correct level is, however, not easy, not least because the permeability of the cane bed is significantly non-uniform.

To facilitate the process of maintaining the derived water level within the cane bed, recessed viewing windows are incorporated into the side walls of the diffuser so that the operators can adjust the flow rates from the water sprays above the cane bed based on their observations of the water levels within the windows. It is not clear, however, if the water levels visible within the windows relate to those within the cane bed and, if they do relate, what the nature of this relationship is.

Extensive and lively discussions during the Study Group focused on developing appropriate mathematical models for the flow of water within the cane bed and the windows. Initially a simple hydrostatic model with gravitational forcing was proposed, however the results obtained from this model appeared to significantly overpredict the water level within the cane bed. The difficulty appeared to be

that the diffusive resistance to water movement was much greater than one might expect using such a model, and so a more detailed model describing the diffusion of water through the cane bed in the spirit of the classical work by Darcy and Richardson was also investigated. This latter model is challenging to solve, but it is hoped that it may still be possible to obtain useful results from one or more appropriately simplified versions of the model.

IMAGE ANALYSIS OF SUGAR CANE PREPARATION

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Executive Summary

Introduction

The process for extraction of sugar from sugarcane in most South African mills involves a counter current washing process in a diffuser. The first step is to break the cane stalk into fine pieces to expose the sugar containing juice for extraction. This preparation of the cane is done using a hammer mill shredder. After preparation, the shredded cane is fed to a diffuser where it is extracted using a counter-current washing process. Best results are achieved with maximum contact between the percolating juice and cane. Too much juice, however, leads to flooding and uncontrolled mixing of the juice with an associated loss in extraction.

The percolation rate in sugarcane diffuser is the volumetric flow rate of juice through a unit area of the cane bed. Percolation performance of the shredded cane in the diffuser depends on the degree of preparation of cane. If the cane is underprepared, it is difficult to wash the sugar from the cane and the over-prepared cane will form a more densely packed bed in the diffuser with low permeability. The permeability influences juice flow patterns in the diffuser and low permeability can cause flooding of the diffuser. Currently, cane preparation is measured using an off-line process. The Study Group suggested the image analysis of the photographs of cane samples for each batch of shredded cane using machine learning techniques to estimate the percolation rate.

Problem definition

Currently there is no continuous automatic measurement of percolation rate in any of the diffusers in sugar factories. Instead, personals use spot checks, which include tracer tests that are periodically performed for the estimation of percolation rate and the fibre packing density within the cane bed density. These are labour intensive and do not account for fluctuations in percolation rate with cane throughput. A continuous monitoring approach for percolation rate estimation and fibre packing density would be an added advantage for the measurement of the juice flow patterns in a diffuser, and could be used for the automatic control of the positioning of sprays in the diffuser to optimise its performance under a wide range of operating conditions. The Study Group used the photographs of shredded cane to estimate the percolation rate using conventional neural network. The estimated percolation can be used as one of the measures to judge the degree of cane preparation and to control the washing process in the diffuser.

Methodology

The analysis of sugarcane shredded images using ML techniques involves the following steps:

1. **Pre-processing**

   Pre-processing is an essential step that is performed in the dataset where all the values have to be transformed into a specific format before feeding it to the machine learning algorithm. It is a mandatory operation that helps in obtaining the needed results.

   In this case, two datasets were provided by the industry representative. The first contains 45 sugar cane images with their laboratory measurements (percolation and density rates). The second dataset contains 120 sugar cane images with their percolation and density rates.

   Different operations were performed before applying the machine learning model on the dataset. These include data merging, categorisation, and data splitting. The following sections describe the process in detail.

2. **Data Merging**

   Data merging refers to the process of combining two or more datasets into one. In this process a shared column between the dataset should exist to use it for the joining operation.

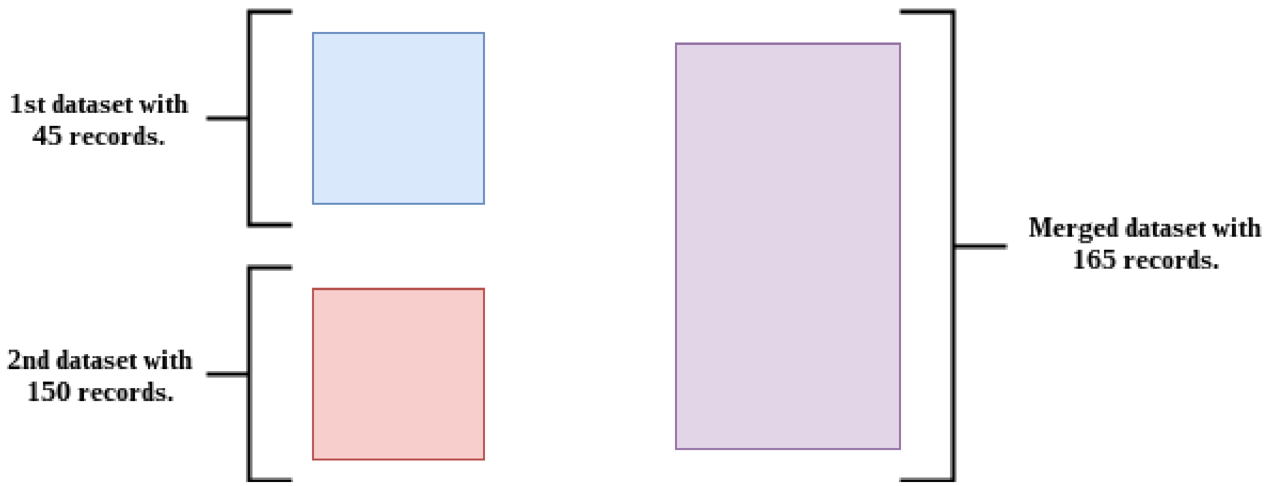

Figure 1: The merging process of the two datasets.

A data merging process was performed on the provided two sugar cane images datasets. The total number of records after merging becomes 165 entities. Every entity has three main features: picture name, percolation and density.

3. **Categorization**

To simplify the process of predicting the percolation rate, a categorisation process was performed, were the numerical values of the percolation were converted into categories. Seven new categories were introduced to the dataset where every sugar cane image falls under one of them. The categories were decided based on the values presented in Table 1. In the table, 0.1 is used as a distance between the categories.

Percolation Density			**Density**		
Start	**End**	**Class**	**Start**	**End**	**Class**
0.7	0.8	1	175	195	1
0.8	0.9	2	195	215	2
0.9	1.0	3	215	235	3
1.0	1.1	4	235	255	4
1.1	1.2	5	255	275	5
1.2	1.3	6	275	295	6
1.3	above	7	295	above	7

Table 1: Percolation and density categories range.

The categorisation process helps in forming discrete values used in the classification problem. The approach that is taken in this problem is to solve it as a classification problem where the sugar cane images fall under one of the 7 classes.

As seen in Figure 2, most of the sugar cane images have percolation rate in the range 0.9 to 1.0 and have density rate in the range 255 to 275.

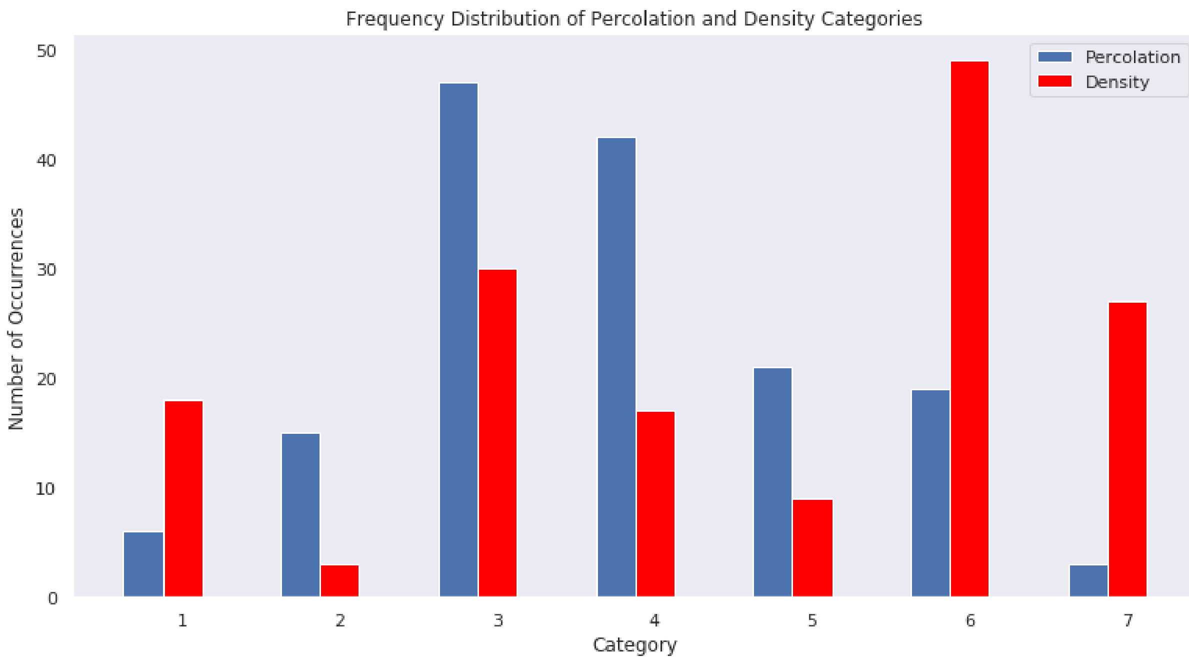

Figure 2: The percolation and density categories and their frequencies.

4. **Exploratory Data Analysis**

Exploratory Data Analysis (EDA) is the process of investigating the data to identify the main characteristics of the data and the relationship between the variables. The EDA process also focuses on identifying outliers, and testing hypothesis. It uses the summary statistics and graphical representations as tools to interpret the data.

All the EDA processes and the code results in this report were developed with Python using Keras, Pandas, Matplotlib and NumPy libraries

Figure 3: The percolation and density rates for the second dataset of sugar cane images.

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https://keras.io, https://pandas.pydata.org, https://matplotlib.org, https://numpy.org

In Figure 3 the values for different sugar cane images are presented with their percolation and density rates. The sugar cane images were taken three times for each group from different angles. This appears in the figure since some multiple images have the same percolation and density rates.

Measure	**Percolation**	**Density**
Count	165.0	165.0
Mean	1.019	261.2
Std	0.141	42.2
Min	0.700	176.7
25%	0.920	232.0
50%	1.000	274.6
75%	1.100	292.7
Max	1.320	332.0

Table 2: Statistical measurements of the percolation and density rates

In Table 2 the statistical measurements for the percolation and density are provided. The total number of available sugar cane images is 165. Percolation rate has smaller range compared to density rate. All the values in the table were rounded off to four digits.

In Figure 4 the number of records that contain missing values were 12 which makes 7.3% of the total available dataset after combining. An interpolation operation could be applied to fill the laboratory measures and recollection could be done for the missing images. However, due to time constrains, these records were removed.

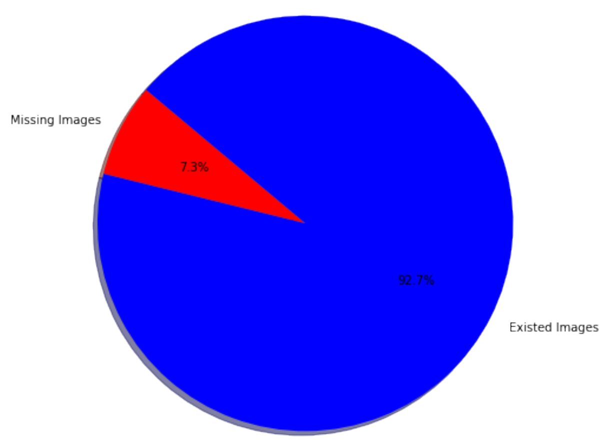

Figure 4: The existed and missing sugar cane images percentages.

In order to check the linearity of the percolation and density rates with respect to the images a distribution plot was used. As seen in Figure 5 the percolation rate is normally distributed, unlike the density rates where all the images for the sugar canes have the likelihood to be higher or lower than the mean value.

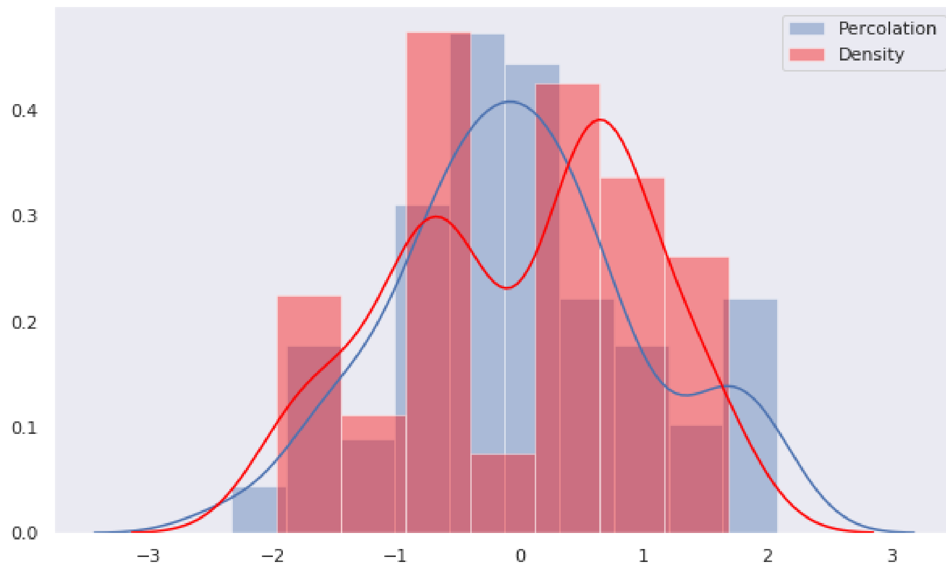

Figure 5: The percolation and density rates distributions.

5. **Other Techniques**

Another avenue that was worth exploring was the reducing of the number of classes. This is justified within the problem constraints since the skilled engineer at the sugar plant only requires a range of values to inform his decisions and not specific values. It is a scientifically sound approach since it reduces the class imbalances that were previously prevalent in the data . By reducing the classes from 7 to 4, the naive model was able to breach the 60% level of test accuracy, with 85% training data split using holdout validation.

Shuffling the data prior to training is also beneficial. There are many reasons why this is considered a good practice. These reasons include reducing variance and ensuring that models remain general and not having over- fit.

In our application, there were pockets of observations clustered and sorted by their class or target value. We shuffle in order to ensure our training/ test/ validation sets are representative of the overall distribution of the data.

GREEN ROOFS TO MITIGATE THE URBAN HEAT ISLAND

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Executive Summary

The urban heat island effect, where a city is a few degrees hotter than the surrounding countryside, is a well-known problem. City planners around the world are investigating strategies to add green spaces to help mitigate this effect. Roofs make up about 20-25% of the urban surface area and the conversion to green roofs is relatively inexpensive, hence conversion may be the most cost effective way of controlling climate change in urban areas.

During the MISG the energy absorbed by typical city surfaces was investigated. In particular the albedo and effect of evaoptranspiration was examined. Key to any analysis of energy accumulation is what happens at the surface. For this the Penman-Monteith equation was considered. This relates the evaporation rate to the amount of solar radiation and atmospheric conditions. It was shown how this relates to a standard surface radiation condition and that there are some differences between the two, with the result that neither is completely adequate for modelling heat transfer at a vegetative surface.

The one dimensional heat equation was analysed for concrete and soil and with different surface conditions (albedo, evapotranspiration). An analytical solution was obtained which could then be integrated to determine the energy within the material (which is released to the atmosphere during the night). It was shown

that, due to its high albedo, a clean concrete roof is actually better at reducing energy absorption than one painted green. Comparing concrete with an albedo of 0.4 against bare soil with an albedo of 0.2, showed that after 4 hours the soil surface could be almost 10 degrees hotter than the concrete. The darker the roof the more heat absorption. However, the energy absorption is significantly reduced when a plant layer is introduced. By adding an evaporation rate of 4mm in 10 hours we found that after 4 hours the soil was some 4 degrees cooler than the concrete. This resulted in a more than 20% reduction in the energy stored in the soil when compared to that in the concrete. Taken over a whole city this reduction makes a huge difference.

The reason that evapotranspiration is so important is that the solar energy is converted to latent heat (turning liquid water into vapour). This conversion of matter from one state to another is very energy intensive, hence the huge reductions in energy absorption. An important effect, which was not included, is that the plant layer shades the heat absorbing surface thus preventing much absorption. There is also an air layer which can take the heat away rather than allow it to be stored for later release. Hence in reality we would expect even greater improvements by converting to green roofs.

In future work we intend to improve the Penman-Monteith equation and correctly incorporate it into the heat flow model. We must also account for finite thickness layers. (Strictly speaking we need to distinguish between roofs thicker or thinner than 50cm, which is the limit of daily variations). Clearly a sensible goal is to verify the work against experiments carried out at the University of the Witwatersrand and also, on a larger scale, by city councils.

MATHEMATICAL MODEL FOR CHESEBOARD WAVES

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Executive Summary

1. The problem

Chessboard waves are often seen near the island of Rhe in France. The pattern is believed to be a cross-sea, and occurs when waves from different directions meet each other at close to right angles. Chessboard waves are dangerous for large ships, boats, swimmers and surfers. The study group was asked to consider the mechanisms that may be the cause of these wave patterns and to derive a mathematical model for them. Images and a video of these waves can be seen here.

2. Early considerations

The group began by considering the local conditions to see if there was some phenomenon that might be the cause, or some unique factor that would create the unusual wave conditions. Using coastal maps we found that the directions of the wind and the current seem to be perpendicular. Topographic maps available on the internet did not include sufficiently small contour intervals to see the local bottom shape. However, it appears that there are oyster farms in the region which suggests a reasonably flat bottom.

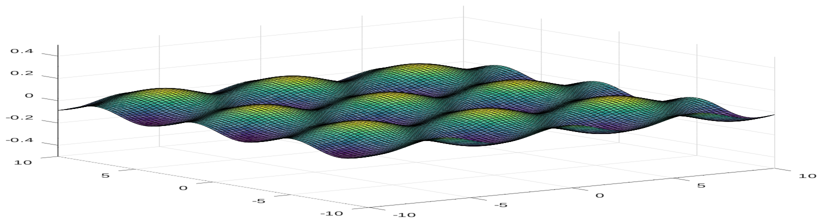

Figure 1: Pattern of intersecting waves assuming sinusoidal, or normal wind-driven waves. This looks nothing like the chessboard waves seen at Isle de Rhe.

Waves fall into several categories that need to be considered. In general waves seen on the ocean are not particularly large and are termed sinusoidal waves, characterised by regular up and down motions. Sinusoidal waves are those that you see approaching the beach, and as they steepen they develop narrower crests and broader troughs. Another class of waves is those termed solitary waves, which consist of a single wave that propagates at a fixed height and travels for long distances undiminished.

3. Which waves?

Figure 1 shows the pattern that would be seen if linear, sinusoidal waves were to cross perpendicular to each other. Clearly this pattern is not what is seen at the location under consideration. On the other hand, a sequence of isolated but separate solitary waves would appear as seen in Figure 2, and this has a great similarity to the waves seen at Isle de Rhe. Therefore the group decided that these are the appropriate waves to be considered in the mathematical model.

A search through the literature found a rich trove of work on solitary waves. This type of wave can be studied by analysing waves that are much longer than they are high. These waves are derived from equations that have names such as Schrödinger's equation (see e.g. [1,5]), as considered in last year's Study Group [2] and the Korteweg-de-Vries (KdV) equation, e.g as described in Grimshaw [3]. Unfortunately, these equations are based on waves travelling in a single direction and not at an angle to each other. It was, as a consequence, not possible within the week to provide a mathematical model to describe the full situation.

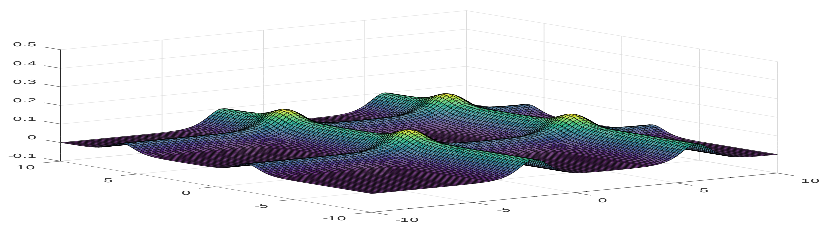

Figure 2: Pattern of intersecting waves assuming solitary waves of KdV form. Although this is not the formal solution to the full two-dimensional problem, it does bear a strong similarity to the waves on Isle de Rhe.

However, it was possible to consider a model for a single train of solitary waves and what may cause them so that the cross-sea behaviour could be inferred as the interaction of such waves travelling from different directions (as suggested by Figure 2).

The KdV equation comes in various forms depending on the situation, but the differences are related to the cause of the waves. The group was able to identify an equation for the case of the propagation of waves over a step in the bottom topography. Figure 3 shows a simulation following the work of Grimshaw et al [4]. Waves propagating to the left are of solitary form, while those travelling to the right are normal ocean waves. In this example, the step rises near zero from zero to one in height and the flow is to the right. This would suggest that one possible mechanism for formation is the surge of waves approaching a shelf in the bottom topography and shedding solitary waves in regular succession. The fact that conditions here consist of the meeting of two seas suggests that a likely mechanism for the formation is either two trains of sinusoidal waves impacting a plateau in the topography from different directions. Whether the ultimate cause turns out to be a plateau or some effect of the meteorological crossing, it would seem this is the appropriate model.

4. Final comments

The chessboard waves observed at Isle de Rhe were considered and the conclusion is that these waves are an interaction of sequences of solitary waves shed by sea swell as it approaches the region from different directions. The regularity of the appearance of such waves suggests that the conditions under which they form also occur regularly. It would therefore seem likely that it is a combination of weather systems with bottom topography that is the instigator of these waves.

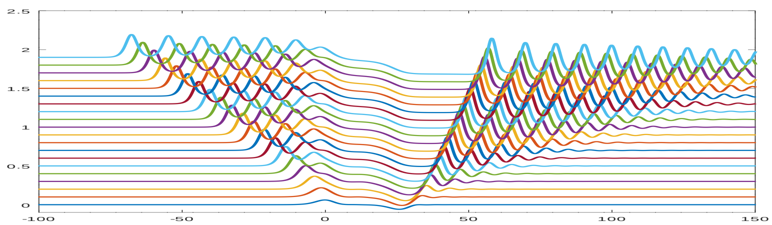

Figure 3: A time sequence of the solution to the KdV equation for flow over a step in the bottom topography using the method of Grimshaw et al [4] Time moves upward as one rises up the figure with an offset of 0.1 units. Waves propagating to the left are of solitary form, while those travelling to the right are normal ocean waves.

A detailed examination of the weather conditions when such events occur and an accurate map of the local bottom topography, as well as more detailed simulations would be required to confirm or refute this suggestion.

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